NON-LINEAR THREE-DIMENSIONAL MODEL OF THE VERTICAL SPRING FORCE OF A TRAMPOLINE BED

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INTRODUCTION

Most of previous papers on biomechanics of trampolining use an ideal spring to calculate the vertical forces acting on an athlete when jumping on a trampoline bed. However due to the mechanical design and fixation of the jumping bed its force-displacement equation is nonlinear.

One paper was found that took this nonlinearity into consideration and which presented a nonlinear **two** dimensional model for calculating the acting forces on an athlete during contact time with the jumping bed (Kraft, 2001).

In this paper a **three dimensional** nonlinear mathematical model is developed to further improve the treatment and description of the nonlinear effects during trampoline jumping. The results of the mathematical calculations were then compared with experimental data from measurements on a trampoline and with data from Kraft (2001).

METHODS

For the development of the mathematical model, we used the force vector diagrams in the cross-sectional view (frontal plane) and longitudinal-sectional view (sagittal plane) of the trampoline bed when loaded with weight. The resultant force vector of the frontal plane and the resultant force vector of the sagittal plane whose lines of action go through the center of the trampoline bed were added together to the total force responsible for the displacement of the trampoline canvas.

At first the nonlinear vertical force-displacement equation $F_V=f(s)$ was theoretically derived as a nonlinear parametric mathematical model with the laws of mechanics. Secondly, to find the parameters and to verify the structure of the equation the force-displacement diagram were statically measured with an experimental setup as follows:

A trampoline which has been used in training was loaded with different weights in steps of 20 kg starting from 0 kg and going up to approximately 400 kg. For each step the vertical displacement, the total length of both centerlines of the jumping bed crosswise and longitudinally were measured as well as the extension of springs and canvas respectively along both centerlines. From that data the graph of the force-displacement relationship was plotted.

The parameters of the mathematical model for the force-displacement relation of the center point of the jumping bed were then calculated with curve fitting procedures. The different size of the contact surfaces with the canvas during foot contact was taken into consideration with a compensative mathematical expression.

RESULTS AND DISCUSSION

The parametric model for the force-displacement relation is a third order polynomial function of the following structure:

$$F_V(s) = c_0 + c_1 s + c_2 s^2 + c_3 s^3$$

The coefficients C_1 and C_3 depend on geometrical dimensions of the trampoline, prestressing of the steel springs and its spring constant. The coefficients $C_0 = C_2 = 0$.

The trampoline we used for experiments was 5,15 m long, 3.08 m wide and 1.03 m high with 13 mm bands. Its vertical force-displacement relationship was calculated as

$$\frac{F_V(s)}{[N]} = -9{,}81 * (372\frac{s}{[m]} + 572\frac{s^3}{[m^3]})$$

The movement differential equation during contact time with the trampoline bed for an athlete doing straight jumps can thus be written with as

$$m\frac{d^2(s)}{dt^2} = F_V(s) - mg$$

which can be solved **explicit analytically** to get the relation of the vertical velocity v of the center of mass m during contact time as function of the vertical displacement s.

SUMMARY

Besides that the studies and simulation carried out by Kraft in (2001) the explicit analytic solution above additionally allows to explicitly calculate the potential energy of the deflected trampoline bed, the kinetic energy of the athlete, upon leaving the canvas and the damping losses of the trampoline during jumps of equal height. We estimated a loss of mechanical energy through damping of about 10% when our athlete was doing successive straight jumps of maximum possible height.

REFERENCES

Kraft, M. (2001) Eine einfache Näherung für die vertikale Federkraft des Trampolintuches.
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